

CSC 550: Introduction to Artificial Intelligence

Fall 2004

Connectionist approach to AI

- neural networks, neuron model
- perceptrons
 - threshold logic, perceptron training, convergence theorem
 - single layer vs. multi-layer
- backpropagation
 - stepwise vs. continuous activation function
- associative memory
 - Hopfield networks
 - parallel relaxation, relaxation as search

1

Symbolic vs. sub-symbolic AI

recall: Good Old-Fashioned AI is inherently symbolic

Physical Symbol System Hypothesis: *A necessary and sufficient condition for intelligence is the representation and manipulation of symbols.*

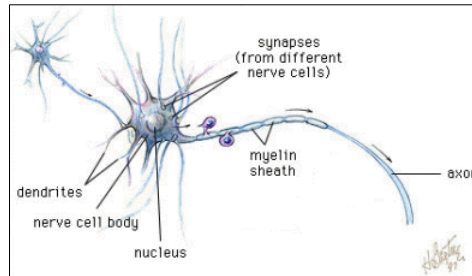
alternatives to symbolic AI

- connectionist models – based on a brain metaphor
 - model individual neurons and their connections
 - properties:* parallel, distributed, sub-symbolic
 - examples:* neural nets, associative memories
- emergent models – based on an evolution metaphor
 - potential solutions compete and evolve
 - properties:* massively parallel,
complex behavior evolves out of simple behavior
 - examples:* genetic algorithms, cellular automata, artificial life

2

Connectionist models (neural nets)

- humans lack the speed & memory of computers
- yet humans are capable of complex reasoning/action
- maybe our brain architecture is well-suited for certain tasks



general brain architecture:

- many (relatively) slow neurons, interconnected
- dendrites serve as input devices (receive electrical impulses from other neurons)
- cell body "sums" inputs from the dendrites (possibly inhibiting or exciting)
- if sum exceeds some threshold, the neuron fires an output impulse along axon

3

Brain metaphor

connectionist models are based on the brain metaphor

- large number of simple, neuron-like processing elements
- large number of weighted connections between neurons
note: the weights encode information, not symbols!
- parallel, distributed control
- emphasis on learning

brief history of neural nets

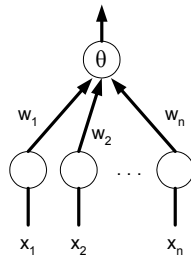
1940's	theoretical birth of neural networks <i>McCulloch & Pitts (1943), Hebb (1949)</i>
1950's & 1960's	optimistic development using computer models <i>Minsky (50's), Rosenblatt (60's)</i>
1970's	DEAD <i>Minsky & Papert showed serious limitations</i>
1980's & 1990's	REBIRTH – new models, new techniques <i>Backpropagation, Hopfield nets</i>

4

Artificial neurons

McCulloch & Pitts (1943) described an artificial neuron

- inputs are either electrical impulse (1) or not (0)
(note: original version used +1 for excitatory and -1 for inhibitory signals)
- each input has a weight associated with it
- the activation function multiplies each input value by its weight
- if the sum of the weighted inputs $\geq \theta$,
then the neuron fires (returns 1), else doesn't fire (returns 0)



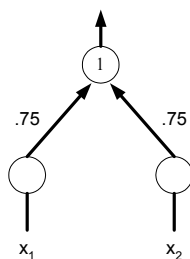
if $\sum w_i x_i \geq \theta$, output = 1
if $\sum w_i x_i < \theta$, output = 0

5

Computation via activation function

can view an artificial neuron as a computational element

- *accepts* or *classifies* an input if the output fires



INPUT: $x_1 = 1, x_2 = 1$

$.75 * 1 + .75 * 1 = 1.5 \geq 1 \rightarrow$ OUTPUT: 1

INPUT: $x_1 = 1, x_2 = 0$

$.75 * 1 + .75 * 0 = .75 < 1 \rightarrow$ OUTPUT: 0

INPUT: $x_1 = 0, x_2 = 1$

$.75 * 0 + .75 * 1 = .75 < 1 \rightarrow$ OUTPUT: 0

INPUT: $x_1 = 0, x_2 = 0$

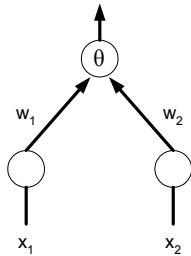
$.75 * 0 + .75 * 0 = 0 < 1 \rightarrow$ OUTPUT: 0

this neuron *computes* the AND function

6

In-class exercise

specify weights and thresholds to compute OR



INPUT: $x_1 = 1, x_2 = 1$

$$w_1 * 1 + w_2 * 1 \geq \theta \quad \rightarrow \text{OUTPUT: 1}$$

INPUT: $x_1 = 1, x_2 = 0$

$$w_1 * 1 + w_2 * 0 \geq \theta \quad \rightarrow \text{OUTPUT: 1}$$

INPUT: $x_1 = 0, x_2 = 1$

$$w_1 * 0 + w_2 * 1 \geq \theta \quad \rightarrow \text{OUTPUT: 1}$$

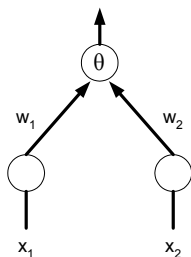
INPUT: $x_1 = 0, x_2 = 0$

$$w_1 * 0 + w_2 * 0 < \theta \quad \rightarrow \text{OUTPUT: 0}$$

7

Another exercise?

specify weights and thresholds to compute XOR



INPUT: $x_1 = 1, x_2 = 1$

$$w_1 * 1 + w_2 * 1 \geq \theta \quad \rightarrow \text{OUTPUT: 0}$$

INPUT: $x_1 = 1, x_2 = 0$

$$w_1 * 1 + w_2 * 0 \geq \theta \quad \rightarrow \text{OUTPUT: 1}$$

INPUT: $x_1 = 0, x_2 = 1$

$$w_1 * 0 + w_2 * 1 \geq \theta \quad \rightarrow \text{OUTPUT: 1}$$

INPUT: $x_1 = 0, x_2 = 0$

$$w_1 * 0 + w_2 * 0 < \theta \quad \rightarrow \text{OUTPUT: 0}$$

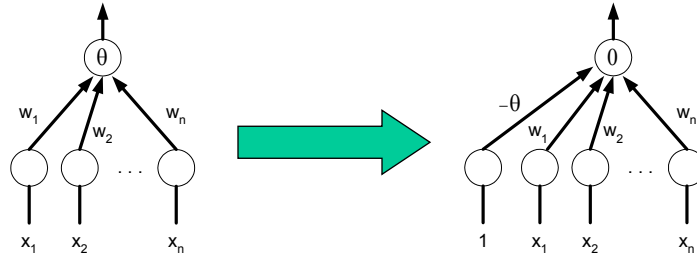
we'll come back to this later...

8

Normalizing thresholds

to make life more uniform, can normalize the threshold to 0

- simply add an additional input $x_0 = 1$, $w_0 = -\theta$



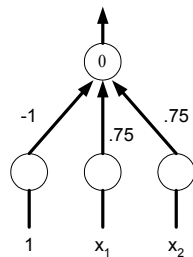
advantage: threshold = 0 for all neurons

$$\sum w_i x_i \geq \theta \quad \equiv \quad -\theta * 1 + \sum w_i x_i \geq 0$$

9

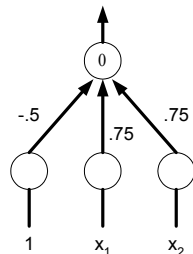
Normalized examples

AND



INPUT: $x_1 = 1, x_2 = 1$	$1 * -1 + .75 * 1 + .75 * 1 = .5 \geq 0$	→ OUTPUT: 1
INPUT: $x_1 = 1, x_2 = 0$	$1 * -1 + .75 * 1 + .75 * 0 = -.25 < 1$	→ OUTPUT: 0
INPUT: $x_1 = 0, x_2 = 1$	$1 * -1 + .75 * 0 + .75 * 1 = -.25 < 1$	→ OUTPUT: 0
INPUT: $x_1 = 0, x_2 = 0$	$1 * -1 + .75 * 0 + .75 * 0 = -1 < 1$	→ OUTPUT: 0

OR



INPUT: $x_1 = 1, x_2 = 1$	$1 * -.5 + .75 * 1 + .75 * 1 = 1 \geq 0$	→ OUTPUT: 1
INPUT: $x_1 = 1, x_2 = 0$	$1 * -.5 + .75 * 1 + .75 * 0 = .25 > 1$	→ OUTPUT: 1
INPUT: $x_1 = 0, x_2 = 1$	$1 * -.5 + .75 * 0 + .75 * 1 = .25 < 1$	→ OUTPUT: 1
INPUT: $x_1 = 0, x_2 = 0$	$1 * -.5 + .75 * 0 + .75 * 0 = -.5 < 1$	→ OUTPUT: 0

10

Perceptrons

Rosenblatt (1958) devised a learning algorithm for artificial neurons

- start with a training set (example inputs & corresponding desired outputs)
- train the network to recognize the examples in the training set (by adjusting the weights on the connections)
- once trained, the network can be applied to new examples

Perceptron learning algorithm:

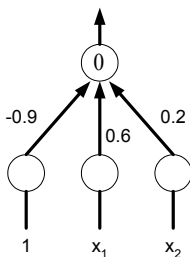
1. Set the weights on the connections with random values.
2. Iterate through the training set, comparing the output of the network with the desired output for each example.
3. If all the examples were handled correctly, then DONE.
4. Otherwise, update the weights for each incorrect example:
 - if should have fired on x_1, \dots, x_n but didn't, $w_i += x_i$ ($0 \leq i \leq n$)
 - if shouldn't have fired on x_1, \dots, x_n but did, $w_i -= x_i$ ($0 \leq i \leq n$)
5. GO TO 2

11

Example: perceptron learning

Suppose we want to train a perceptron to compute AND

training set: $x_1 = 1, x_2 = 1 \rightarrow 1$
 $x_1 = 1, x_2 = 0 \rightarrow 0$
 $x_1 = 0, x_2 = 1 \rightarrow 0$
 $x_1 = 0, x_2 = 0 \rightarrow 0$



randomly, let: $w_0 = -0.9, w_1 = 0.6, w_2 = 0.2$

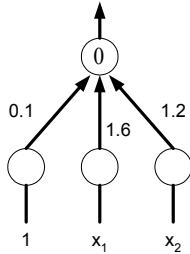
using these weights:

$x_1 = 1, x_2 = 1:$	$-0.9*1 + 0.6*1 + 0.2*1$	$= -0.9 \rightarrow 0$	WRONG
$x_1 = 1, x_2 = 0:$	$-0.9*1 + 0.6*1 + 0.2*0$	$= -0.3 \rightarrow 0$	OK
$x_1 = 0, x_2 = 1:$	$-0.9*1 + 0.6*0 + 0.2*1$	$= -0.7 \rightarrow 0$	OK
$x_1 = 0, x_2 = 0:$	$-0.9*1 + 0.6*0 + 0.2*0$	$= -0.9 \rightarrow 0$	OK

new weights: $w_0 = -0.9 + 1 = 0.1$
 $w_1 = 0.6 + 1 = 1.6$
 $w_2 = 0.2 + 1 = 1.2$

12

Example: perceptron learning (cont.)



using these updated weights:

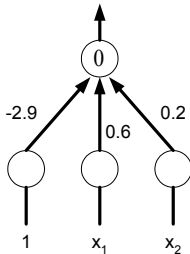
$x_1 = 1, x_2 = 1:$	$0.1*1 + 1.6*1 + 1.2*1$	$= 2.9 \rightarrow 1$	OK
$x_1 = 1, x_2 = 0:$	$0.1*1 + 1.6*1 + 1.2*0$	$= 1.7 \rightarrow 1$	WRONG
$x_1 = 0, x_2 = 1:$	$0.1*1 + 1.6*0 + 1.2*1$	$= 1.3 \rightarrow 1$	WRONG
$x_1 = 0, x_2 = 0:$	$0.1*1 + 1.6*0 + 1.2*0$	$= 0.1 \rightarrow 1$	WRONG

new weights:

$$w_0 = 0.1 - 1 - 1 - 1 = -2.9$$

$$w_1 = 1.6 - 1 - 0 - 0 = 0.6$$

$$w_2 = 1.2 - 0 - 1 - 0 = 0.2$$



using these updated weights:

$x_1 = 1, x_2 = 1:$	$-2.9*1 + 0.6*1 + 0.2*1$	$= -2.1 \rightarrow 0$	WRONG
$x_1 = 1, x_2 = 0:$	$-2.9*1 + 0.6*1 + 0.2*0$	$= -2.3 \rightarrow 0$	OK
$x_1 = 0, x_2 = 1:$	$-2.9*1 + 0.6*0 + 0.2*1$	$= -2.7 \rightarrow 0$	OK
$x_1 = 0, x_2 = 0:$	$-2.9*1 + 0.6*0 + 0.2*0$	$= -2.9 \rightarrow 0$	OK

new weights:

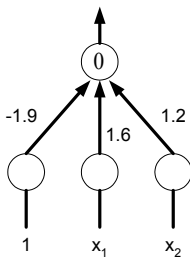
$$w_0 = -2.9 + 1 = -1.9$$

$$w_1 = 0.6 + 1 = 1.6$$

$$w_2 = 0.2 + 1 = 1.2$$

13

Example: perceptron learning (cont.)



using these updated weights:

$x_1 = 1, x_2 = 1:$	$-1.9*1 + 1.6*1 + 1.2*1$	$= 0.9 \rightarrow 1$	OK
$x_1 = 1, x_2 = -1:$	$-1.9*1 + 1.6*1 + 1.2*0$	$= -1.5 \rightarrow 0$	OK
$x_1 = -1, x_2 = 1:$	$-1.9*1 + 1.6*0 + 1.2*1$	$= -2.3 \rightarrow 0$	OK
$x_1 = -1, x_2 = -1:$	$-1.9*1 + 1.6*0 + 1.2*0$	$= -4.7 \rightarrow 0$	OK

DONE!

EXERCISE: train a perceptron to compute OR

14

Convergence

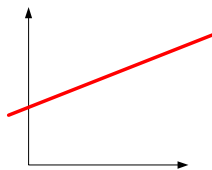
key reason for interest in perceptrons:

Perceptron Convergence Theorem

The perceptron learning algorithm will always find weights to classify the inputs if such a set of weights exists.

Minsky & Papert showed weights exist if and only if the problem is *linearly separable*

intuition: consider the case with 2 inputs, x_1 and x_2



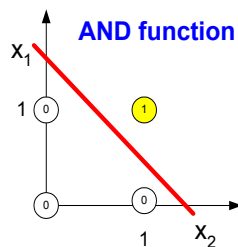
if you can draw a line and separate the accepting & non-accepting examples, then *linearly separable*

the intuition generalizes: for n inputs, must be able to separate with an $(n-1)$ -dimensional plane.

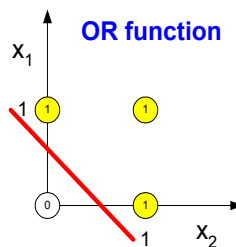
see <http://www.avaye.com/index.php/neuralnets/simulators/freeware/perceptron>

15

Linearly separable



AND function



OR function

why does this make sense?

firing depends on $w_0 + w_1x_1 + w_2x_2 \geq 0$

border case is when $w_0 + w_1x_1 + w_2x_2 = 0$

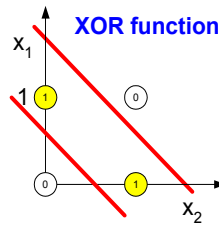
i.e., $x_2 = (-w_1/w_2)x_1 + (-w_0/w_2)$ the equation of a line

the training algorithm simply shifts the line around (by changing the weight) until the classes are separated

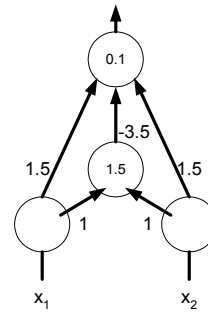
16

Inadequacy of perceptrons

inadequacy of perceptrons is due to the fact that many simple problems are not linearly separable



however, can compute XOR by introducing a new, hidden unit



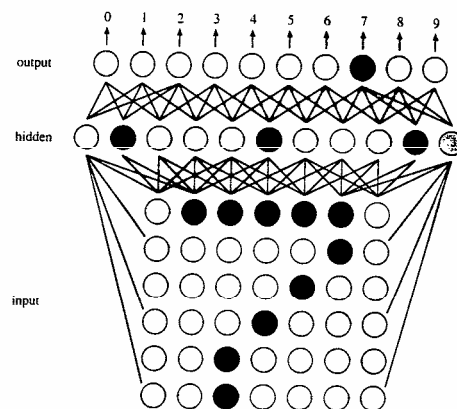
17

Hidden units

the addition of hidden units allows the network to develop complex feature detectors (i.e., internal representations)

e.g., Optical Character Recognition (OCR)

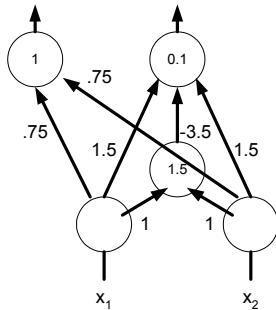
- perhaps one hidden unit "looks for" a horizontal bar
- another hidden unit "looks for" a diagonal
- another looks for the vertical base
- the combination of specific hidden units indicates a 7



18

Building multi-layer nets

smaller example: can combine perceptrons to perform more complex computations (or classifications)



3-layer neural net

2 input nodes
1 hidden node
2 output nodes

RESULT?

HINT: left output node is AND
right output node is XOR

HALF ADDER

19

Hidden units & learning

every classification problem has a perceptron solution if enough hidden layers are used

- i.e., multi-layer networks can compute anything (recall: can simulate AND, OR, NOT gates)

expressiveness is not the problem – learning is!

- it is not known how to systematically find solutions
- the Perceptron Learning Algorithm can't adjust weights between levels

Minsky & Papert's results about the "inadequacy" of perceptrons pretty much killed neural net research in the 1970's

rebirth in the 1980's due to several developments

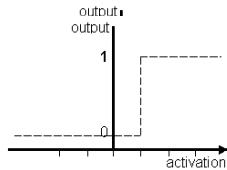
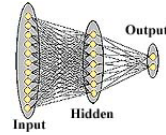
- faster, more parallel computers
- new learning algorithms e.g., backpropagation
- new architectures e.g., Hopfield nets

20

Backpropagation nets

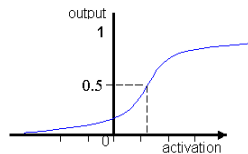
backpropagation nets are multi-layer networks

- normalize inputs between 0 (inhibit) and 1 (excite)
- utilize a continuous activation function



perceptrons utilize a stepwise activation function

$$\text{output} = \begin{cases} 1 & \text{if sum} \geq 0 \\ 0 & \text{if sum} < 0 \end{cases}$$



backpropagation nets utilize a continuous activation function

$$\text{output} = 1 / (1 + e^{-\text{sum}})$$

21

Backpropagation example (XOR)

$$x_1 = 1, x_2 = 1$$

$$\text{sum}(H_1) = -2.2 + 5.7 + 5.7 = 9.2, \text{ output}(H_1) = 0.99$$

$$\text{sum}(H_2) = -4.8 + 3.2 + 3.2 = 1.6, \text{ output}(H_2) = 0.83$$

$$\text{sum} = -2.8 + (0.99 * 6.4) + (0.83 * -7) = -2.28, \text{ output} = 0.09$$

$$x_1 = 1, x_2 = 0$$

$$\text{sum}(H_1) = -2.2 + 5.7 + 0 = 3.5, \text{ output}(H_1) = 0.97$$

$$\text{sum}(H_2) = -4.8 + 3.2 + 0 = -1.6, \text{ output}(H_2) = 0.17$$

$$\text{sum} = -2.8 + (0.97 * 6.4) + (0.17 * -7) = 2.22, \text{ output} = 0.90$$

$$x_1 = 0, x_2 = 1$$

$$\text{sum}(H_1) = -2.2 + 0 + 5.7 = 3.5, \text{ output}(H_1) = 0.97$$

$$\text{sum}(H_2) = -4.8 + 0 + 3.2 = -1.6, \text{ output}(H_2) = 0.17$$

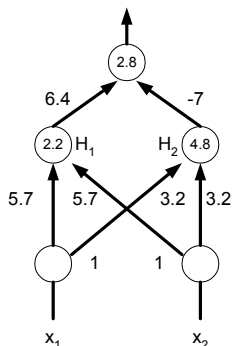
$$\text{sum} = -2.8 + (0.97 * 6.4) + (0.17 * -7) = 2.22, \text{ output} = 0.90$$

$$x_1 = 0, x_2 = 0$$

$$\text{sum}(H_1) = -2.2 + 0 + 0 = -2.2, \text{ output}(H_1) = 0.10$$

$$\text{sum}(H_2) = -4.8 + 0 + 0 = -4.8, \text{ output}(H_2) = 0.01$$

$$\text{sum} = -2.8 + (0.10 * 6.4) + (0.01 * -7) = -2.23, \text{ output} = 0.10$$



22

Backpropagation learning

there exists a systematic method for adjusting weights, but no global convergence theorem (as was the case for perceptrons)

backpropagation (backward propagation of error) – vaguely stated

- select arbitrary weights
- pick the first test case
- make a forward pass, from inputs to output
- compute an error estimate and make a backward pass, adjusting weights to reduce the error
- repeat for the next test case

testing & propagating for all training cases is known as an *epoch*

despite the lack of a convergence theorem, backpropagation works well in practice

- however, many epochs may be required for convergence

23

Backpropagation example

consider the following political poll, taken by six potential voters

- each ranked various topics as to their importance, scale of 0 to 10
- voters 1-3 identified themselves as Republicans, voters 4-6 as Democrats

	Budget	Defense	Crime	Environment	Social Security
voter 1	9	6	8	3	1
voter 2	8	8	4	6	4
voter 3	7	2	4	6	3
voter 4	5	5	8	4	8
voter 5	3	1	6	8	8
voter 6	6	3	4	3	6

based on survey responses, can we train a neural net to recognize Republicans and Democrats?

24

Backpropagation example (cont.)

utilize the neural net (backpropagation) simulator at:

<http://www.cs.ubc.ca/labs/lci/CIspace/Version4/neural/>

note: inputs to network can be real values between -1.0 and 1.0

- in this example, can use fractions to indicate the range of survey responses
e.g., response of 8 \rightarrow input value of 0.8
- **APPLET IS FLAKEY - BE CAREFUL AND SPECIFY ALL INPUT/OUTPUT VALUES**

make sure you recognize the training set accurately.

- how many training cycles are needed?
- how many hidden nodes?

25

Backpropagation example (cont.)

using the neural net, try to classify the following new respondents

	Budget	Defense	Crime	Environment	Social Security
voter 1	9	6	8	3	1
voter 2	8	8	4	6	4
voter 3	7	2	4	6	3
voter 4	5	5	8	4	8
voter 5	3	1	6	8	8
voter 6	6	3	4	3	6
voter 7	10	10	10	1	1
voter 8	5	2	2	7	7
voter 9	8	3	3	3	8

26

HW 5 exercise

for HW 5, you will design a neural net for student advising

- ask (at least 5) questions about interests, aptitudes, lifestyle goals
e.g., Do you like to read books? play video games? work with people?
- differentiate between (at least 3) different majors
e.g., history (0.0) – CS (0.5) – math (1.0)
- train on a set of peers (at least 2 people per major – the more the better)
- test on at least 1 person per major

27

Problems/challenges in neural nets research

learning problem

- can the network be trained to solve a given problem?
- if not linearly separable, no guarantee (but backpropagation is effective in practice)

architecture problem

- are there useful architectures for solving a given problem?
- most applications use a 3-layer (input, hidden, output), fully-connected net

scaling problem

- how can training time be minimized?
- difficult/complex problems may require thousands of epochs

generalization problem*

- how know if the trained network will behave "reasonably" on new inputs?

*example from AI video: backpropagation net trained to identify tanks in photos
trained on both positive and negative examples, very effective
when tested on new photos, failed miserably
WHY?*

28

Generalization problem

suppose a network is trained to recognize digits:

- training set for 1:

1	1	1	1
---	---	---	---
- training set for 2:

2	2	2	2
---	---	---	---

when the network is asked to identify:

2

 it comes back with 1. WHY?

there is always a danger that the network will focus on specific features as opposed to general patterns (especially if many hidden nodes ?)

to avoid networks that are too specific, *cross-validation* is often used

1. split training set into training & validation data
2. after each epoch, test the net on the validation data
3. continue until performance on the validation data diminishes (e.g., hillclimb)

29

Neural net applications

pattern classification

- 9 of top 10 US credit card companies use Falcon
uses neural nets to model customer behavior, identify fraud
claims improvement in fraud detection of 30-70%
- Sharp, Mitsubishi, ... -- Optical Character Recognition (OCR)
(see <http://www.sund.de/netze/applets/BPN/bpn2/ochre.html>)

prediction & financial analysis

- Merrill Lynch, Citibank, ... -- financial forecasting, investing
- Spiegel – marketing analysis, targeted catalog sales

control & optimization

- Texaco – process control of an oil refinery
- Intel – computer chip manufacturing quality control
- AT&T – echo & noise control in phone lines (filters and compensates)
- Ford engines utilize neural net chip to diagnose misfirings, reduce emissions
- also from AI video: ALVINN project at CMU trained a neural net to drive
backpropagation network: video input, 9 hidden units, 45 outputs

30

Interesting variation: Hopfield nets

in addition to uses as acceptor/classifier, neural nets can be used as associative memory – Hopfield (1982)

- can store multiple patterns in the network, retrieve

interesting features

- distributed representation
 - info is stored as a pattern of activations/weights
 - multiple info is imprinted on the same network
- content-addressable memory
 - store patterns in a network by adjusting weights
 - to retrieve a pattern, specify a portion (will find a near match)
- distributed, asynchronous control
 - individual processing elements behave independently
- fault tolerance
 - a few processors can fail, and the network will still work

31

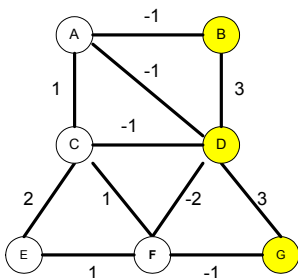
Hopfield net examples

processing units are in one of two states: *active* or *inactive*

units are connected with weighted, symmetric connections

positive weight → excitatory relation

negative weight → inhibitory relation



to imprint a pattern

- adjust the weights appropriately (no general algorithm is known, basically ad. hoc)

to retrieve a pattern:

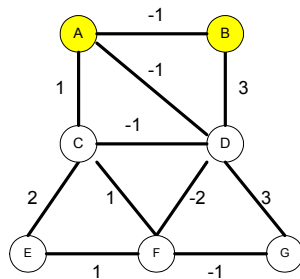
- specify a partial pattern in the net
- perform *parallel relaxation* to achieve a steady state representing a near match

32

Parallel relaxation

parallel relaxation algorithm:

1. pick a random unit
2. sum the weights on connections to active neighbors
3. if the sum is positive \rightarrow make the unit active
if the sum is negative \rightarrow make the unit inactive
4. repeat until a stable state is achieved



this Hopfield net has 4 stable states

- what are they?
- parallel relaxation will start with an initial state and converge to one of these stable states

33

Why does it converge?

parallel relaxation is guaranteed to converge on a stable state in a finite number of steps (i.e., node state flips)

WHY?

Define $H(\text{net}) = \sum (\text{weights connecting active nodes})$

Theorem: Every step in parallel relaxation increases $H(\text{net})$.

If step involves making a node active, this is because the sum of weights to active neighbors > 0 . Therefore, making this node active increases $H(\text{net})$.

If step involves making a node inactive, this is because the sum of the weights to active neighbors < 0 . Therefore, making this node active increases $H(\text{net})$.

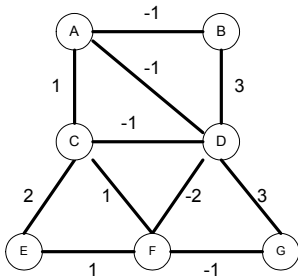
Since $H(\text{net})$ is bounded, relaxation must eventually stop \rightarrow stable state

34

Hopfield nets in Scheme

need to store the Hopfield network in a Scheme structure

- could be unstructured, graph = collection of edges
- could structure to make access easier



```
(define HOPFIELD-NET
  '((A (B -1) (C 1) (D -1))
    (B (A -1) (D 3))
    (C (A 1) (D -1) (E 2) (F 1))
    (D (A -1) (B 3) (C -1) (E -2) (G 3))
    (E (C 2) (F 1))
    (F (C 1) (D -2) (E 1) (G -1))
    (G (D 3) (F -1))))
```

35

Parallel relaxation in Scheme

```
(define (relax active)

  (define (neighbor-sum neighbors active)
    (cond ((null? neighbors) 0)
          ((member (caar neighbors) active)
           (+ (cadar neighbors) (neighbor-sum (cdr neighbors) active)))
          (else (neighbor-sum (cdr neighbors) active))))

  (define (get-unstables net active)
    (cond ((null? net) '())
          ((and (member (caar net) active) (< (neighbor-sum (cdar net) active) 0))
           (cons (caar net) (get-unstables (cdr net) active)))
          ((and (not (member (caar net) active))
                (> (neighbor-sum (cdar net) active) 0))
           (cons (caar net) (get-unstables (cdr net) active)))
          (else (get-unstables (cdr net) active))))

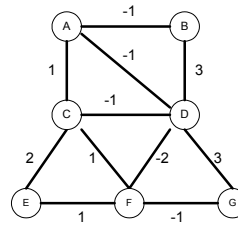
  (let ((unstables (get-unstables HOPFIELD-NET active)))
    (if (null? unstables)
        active
        (let ((selected (list-ref unstables (random (length unstables)))))
          (if (member selected active)
              (relax (remove selected active))
              (relax (cons selected active))))))))
```

36

Relaxation examples

```
> (relax '())  
( )  
  
> (relax '(b d g))  
(b d g)  
  
> (relax '(a c e f))  
(a c e f)  
  
> (relax '(b c d e g))  
(b c d e g)
```

```
> (relax '(a b))  
(g d b)  
  
> (relax '(a b c e f))  
(a c e f)  
  
> (relax '(a b c d e f g))  
(b c d e g)  
  
> (relax '(a b c d))  
(e g b c d)  
  
> (relax '(d c b a))  
(g d b)
```



parallel relaxation will identify stored patterns (since stable)

if you input a partial pattern, parallel relaxation will converge on a stored pattern

- what can you say about the stored pattern that is reached?
- is it in some sense the "closest" match?

37

Associative memory

a Hopfield net is associative memory

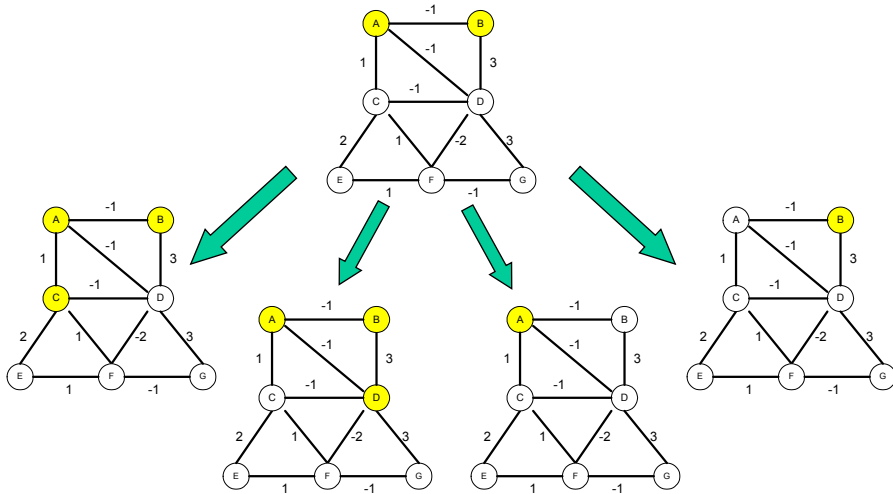
- patterns are stored in the network via weights
 - if presented with a stored pattern, relaxation will verify its presence in the net
 - if presented with a new pattern, relaxation will find a match in the net
- if unstable nodes are selected at random, can't make any claims of closeness
- ideally, we would like to find the "closest" or "best" match
- fewest differences in active nodes?*
- fewest flips between states?*

38

Parallel relaxation as search

can view the parallel relaxation algorithm as search

- state is a list of active nodes
- moves are obtained by flipping an unstable neighbor state



39

Parallel relaxation using BFS

could use breadth first search (BFS) to find the pattern that is the fewest number of flips away from input pattern

```
(define (relax active)
  (car (bfs-nocycles active)))

(define (GET-MOVES active)
  (define (get-moves-help unstables)
    (cond ((null? unstables) '())
          ((member (car unstables) active)
           (cons (remove (car unstables) active)
                 (get-moves-help (cdr unstables))))
          (else (cons (cons (car unstables) active)
                      (get-moves-help (cdr unstables))))))
  (get-moves-help (get-unstables HOPFIELD-NET active)))

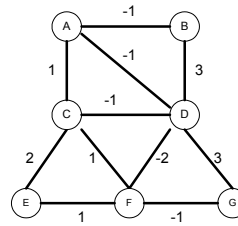
(define (GOAL? active)
  (null? (get-unstables HOPFIELD-NET active)))
```

40

Relaxation examples

```
> (relax '())  
( )  
  
> (relax '(b d g))  
(b d g)  
  
> (relax '(a c e f))  
(a c e f)  
  
> (relax '(b c d e g))  
(b c d e g)
```

```
> (relax '(a b))  
(g d b)  
  
> (relax '(a b c e f))  
(a c e f)  
  
> (relax '(a b c d e f g))  
(b c d e g)  
  
> (relax '(a b c d))  
(g b d)  
  
> (relax '(d c b a))  
(g d b)
```



parallel relaxation will identify stored patterns (since stable)

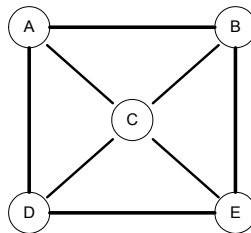
if you input a partial pattern, parallel relaxation will converge on "closest" pattern

41

Another example

consider the following Hopfield network

- specify weights that would store the following patterns: AD, BE, ACE



42

Next week...

Emergent models of machine learning

- genetic algorithms
- cellular automata
- artificial life

Read Chapter 11

Be prepared for a quiz on

- this week's lecture (moderately thorough)
- the reading (superficial)