CSC 321: Data Structures

Fall 2018

Hash tables

- HashSet & HashMap
- hash table, hash function
- collisions
  - ➢ linear probing, lazy deletion, clustering, rehashing
  - ➢ chaining
- Java hashCode method

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HashSet & HashMap

recall: `TreeSet` & `TreeMap` use an underlying binary search tree (actually, a red-black tree) to store values

- as a result, add/put, contains/get, and remove are O(log N) operations
- iteration over the Set/Map can be done in O(N)

the other implementations of the `Set` & `Map` interfaces, `HashSet` & `HashMap`, use a "magic" data structure to provide O(1) operations*

**legal disclaimer:* performance can degrade to O(N) under bad/unlikely conditions
however, careful setup and maintenance can deliver O(1) in practice

the underlying data structure is known as a *Hash Table*

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Hash tables

a hash table is a data structure that supports *constant time* insertion, deletion, and search *on average*

- degenerative performance is possible, but unlikely
- it may waste some storage
- iteration order is not defined (and may even change over time)

idea: data items are stored in a table, based on a key

- the key is mapped to an index in the table, where the data is stored/accessed

example: letter frequency

- want to count the number of occurrences of each letter in a file
- have an array of 26 counters, map each letter to an index
- to count a letter, map to its index and increment

"A" → 0	1
"B" → 1	0
"C" → 2	3
	. . .
"Z" → 25	0

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Mapping examples

extension: word frequency

- must map entire words to indices, e.g.,

"A" → 0	"AA" → 26	"BA" → 52	. . .
"B" → 1	"AB" → 27	"BB" → 53	. . .
. . .	. . .	. . .	
"Z" → 25	"AZ" → 51	"BZ" → 77. . .	

- PROBLEM?

mapping each potential item to a unique index is generally not practical

\# of 1 letter words = 26
\# of 2 letter words = 26^2 = 676
\# of 3 letter words = 26^3 = 17,576
. . .

- even if you limit words to at most 8 characters, need a table of size 217,180,147,158
- for any given file, the table will be mostly empty!

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Table size < data range

since the actual number of items stored is generally MUCH smaller than the number of potential values/keys:

- can have a smaller, more manageable table

  e.g., table size = 26
  possible mapping: map word based on first letter

  "A*" → 0 "B*" → 1 . . . "Z*" → 25

  e.g., table size = 1000
  possible mapping: add ASCII values of letters, mod by 1000

  "AB" → 65 + 66 = 131

  "BANANA" → 66 + 65 + 78 + 65 + 78 + 65 = 417

  "BANANABANANABANANA" → 417 + 417 + 417 = 1251 % 1000 = 251

- POTENTIAL PROBLEMS?

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Collisions

the mapping from a key to an index is called a *hash function*

- the hash function can be written independent of the table size
- if it maps to an index > table size, simply wrap-around (i.e., index % tableSize)

since `|range(hash function)| < |domain(hash function)|`,
Pigeonhole Principle ensures *collisions* are possible (v_1 & v_2 → same index)

"ACT" → 67 + 65 + 84 = 216 "CAT" → 67 + 65 + 84 = 216

techniques exist for handling collisions, but they are costly (LATER)

it's best to avoid collisions as much as possible – HOW?

- want to be sure that the hash function distributes the key evenly

- e.g., "sum of ASCII codes" hash function
  OK if table size is 1000
  BAD if table size is 10,000
  most words are ≤ 10 letters, so max sum of ASCII codes = 1,270
  so most entries are mapped to first 13% of table

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Better hash function

a good hash function for words should

- produce an even spread, regardless of table size
- take order of letters into account (to handle anagrams)

- the hash function used by `java.util.String` multiplies the ASCII code for each character by a power of 31

$$\texttt{hashCode()} = char_0*31^{(len-1)} + char_1*31^{(len-2)} + char_2*31^{(len-3)} + \ldots + char_{(len-1)}$$

where len = `this.length()`, $char_i$ = `this.charAt(i)`:

```
/**
 * Hash code for java.util.String class
 *   @return an int used as the hash index for this string
 */
private int hashCode() {
    int hashIndex = 0;

    for (int i = 0; i < this.length(); i++) {
        hashIndex = (hashIndex*31 + this.charAt(i));
    }
    return hashIndex;
}
```

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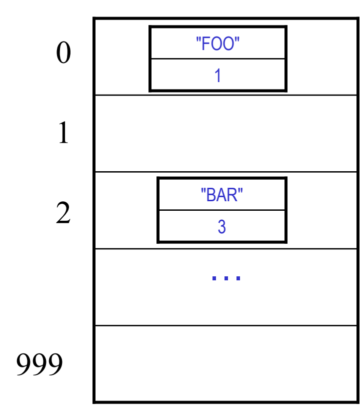
Word frequency example

returning to the word frequency problem

- pick a hash function
- pick a table size

- store word & associated count in the table

- as you read in words,
  map to an index using the hash function
  if an entry already exists, increment
  otherwise, create entry with count = 1



WHAT ABOUT COLLISIONS?

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Linear probing

linear probing is a simple strategy for handling collisions

- if a collision occurs, try next index & keep looking until an empty one is found (wrap around to the beginning if necessary)

example: assume "first letter" hash function

- insert "BOO", "BAR", "COO", "BOW, …

0	
1	
2	
3	
4	
	…
25	

linear probing requires "lazy deletion"

- when you delete an item, you can't just empty the location, since it would leave a hole
- subsequent searches would reach that whole and stop probing
- instead, leave a marker (a.k.a a tombstone) in that spot 0 can be overwritten but not skipped when probing

example: given above insertions

- delete "BAR", search for "COO"

Clustering and load factor

0	
1	"BOO"
2	"BIZ"
3	"COO"
4	"DOG"
5	
6	
7	

in practice, probes are not independent

- as the table fills, clusters appear that degrade performance

maps to	0, 5-7 require	1 check
map to	4 requires	2 checks
map to	3 requires	3 checks
map to	2 requires	4 checks
map to	1 requires	5 checks

average = 18/8 = 2.25 checks

the *load factor* λ is the fraction of the table that is full

empty table $\lambda = 0$ half full table $\lambda = 0.5$ full table $\lambda = 1$

THEOREM: assuming a reasonably large table, the average number of locations examined per insertion is roughly $(1 + 1/(1-\lambda)^2)/2$

empty table	$(1 + 1/(1 - 0)^2)/2 = 1$
half full	$(1 + 1/(1 – .5)^2)/2 = 2.5$
3/4 full	$(1 + 1/(1 - .75)^2)/2 = 8.5$
9/10 full	$(1 + 1/(1 - .9)^2)/2 = 50.5$

Rehashing

as long as you keep the load factor low (e.g., < 0.75), inserting, deleting and searching a hash table are all O(1) operations

if the table becomes too full, then must resize

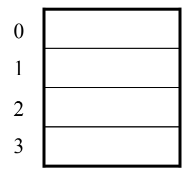
- create new table at least twice as big
- just copy over table entries to same locations???
- NO! when you resize, you have to rehash existing entries
  new table size → new hash function (+ different wraparound)

LET hashCode = word.length()

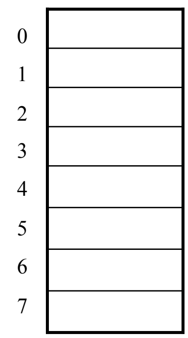
ADD "UP"

ADD "OUT"

ADD "YELLOW"



NOW
RESIZE
AND
REHASH



Chaining

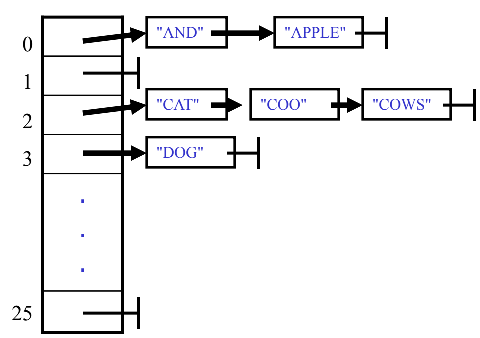
linear probing (or variants) were initially used when memory was expensive

- clustering, lazy deletion, and rehashing are all issues

modern languages like Java utilize a different approach

chaining:

- each entry in the hash table is a bucket (list)
- when you add an entry, hash to correct index then add to bucket
- when you search for an entry, hash to correct index then search sequentially



Analysis of chaining

in practice, chaining is generally faster than probing

- cost of insertion is O(1) – simply map to index and add to list

- cost of search is proportional to number of items already mapped to same index
  e.g., using naïve "first letter" hash function, searching for "APPLE" might requires traversing a list of all words beginning with 'A'

  if hash function is fair, then average size of each bucket is λ (load factor)
  → average cost of a successful search is roughly $\lambda/2$

chaining is sensitive to the load factor, but not as much as probing – WHY?

chaining uses more memory – WHY?

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Hashtable class

java.util
Class Hashtable<K,V>

Constructor Summary
Hashtable() Constructs a new, empty hashtable with a default initial capacity (11) and load factor (0.75).
Hashtable(int initialCapacity) Constructs a new, empty hashtable with the specified initial capacity and default load factor (0.75).
Hashtable(int initialCapacity, float loadFactor) Constructs a new, empty hashtable with the specified initial capacity and the specified load factor.
Hashtable(Map<? extends K,? extends V> t) Constructs a new hashtable with the same mappings as the given Map.

Method Summary	
void	clear() Clears this hashtable so that it contains no keys.
Object	clone() Creates a shallow copy of this hashtable.
boolean	contains(Object value) Tests if some key maps into the specified value in this hashtable.
boolean	containsKey(Object key) Tests if the specified object is a key in this hashtable.
boolean	containsValue(Object value) Returns true if this hashtable maps one or more keys to this value.
Enumeration<V>	elements() Returns an enumeration of the values in this hashtable.
Set<Map.Entry<K,V>>	entrySet() Returns a Set view of the mappings contained in this map.
boolean	equals(Object o) Compares the specified Object with this Map for equality, as per the definition in the Map interface.
V	get(Object key) Returns the value to which the specified key is mapped, or null if this map contains no mapping for the key.
int	hashCode() Returns the hash code value for this Map as per the definition in the Map interface.
boolean	isEmpty() Tests if this hashtable maps no keys to values.
Enumeration<K>	keys() Returns an enumeration of the keys in this hashtable.
Set<K>	keySet() Returns a Set view of the keys contained in this map.
V	put(K key, V value) Maps the specified key to the specified value in this hashtable.
void	putAll(Map<? extends K,? extends V> t) Copies all of the mappings from the specified map to this hashtable.
protected void	rehash() Increases the capacity of and internally reorganizes this hashtable, in order to accommodate and access its entries more efficiently.
V	remove(Object key) Removes the key (and its corresponding value) from this hashtable.
int	size() Returns the number of keys in this hashtable.
String	toString() Returns a string representation of this Hashtable object in the form of a set of entries, enclosed in braces and separated by the ASCII characters ", " (comma and space).

Java provides a basic hash table implementation

- utilizes chaining
- can specify the initial table size & threshold for load factor
- can even force a rehashing

not commonly used, instead provides underlying structure for HashSet & HashMap

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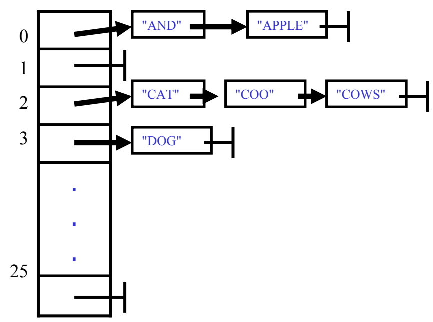
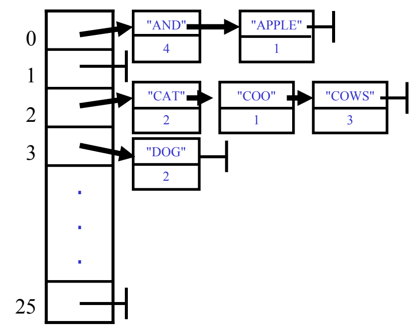
HashSet & HashMap

`java.util.HashSet` and `java.util.HashMap` use hash table w/ chaining

- e.g., `HashSet<String>` `HashMap<String, Integer>`





- defaults: table size = 16, max capacity before rehash = 75%
  can override these defaults in the HashSet/HashMap constructor call

note: iterating over a HashSet or HashMap is: O(num stored + table size) WHY?

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Word frequencies (again)

using HashMap instead of TreeMap

- containsKey, get & put operations are all O(1)*
- however, iterating over the keySet (and their values) does not guarantee any order

- if you really care about speed → use HashSet/HashMap
- if the data/keys are comparable & order matters → use TreeSet/TreeMap

```
import java.util.Map;
import java.util.HashMap;
import java.util.Scanner;
import java.io.File;

public class WordFreq {
    private Map<String, Integer> words;

    public WordFreq() {
        words = new HashMap<String, Integer>();
    }

    public WordFreq(String filename) {
        this();
        try {
            Scanner infile = new Scanner(new File(filename));
            while (infile.hasNext()) {
                String nextWord = infile.next();
                this.add(nextWord);
            }
        }
        catch (java.io.FileNotFoundException e) {
            System.out.println("FILE NOT FOUND");
        }
    }

    public void add(String newWord) {
        String cleanWord = newWord.toLowerCase();
        if (words.containsKey(cleanWord)) {
            words.put(cleanWord, words.get(cleanWord)+1);
        }
        else {
            words.put(cleanWord, 1);
        }
    }

    public void showAll() {
        for (String str : words.keySet()) {
            System.out.println(str + ": " + words.get(str));
        }
    }
}
```

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hashCode function

```
import java.util.Calendar;
import java.util.GregorianCalendar;

public class Person {
    private String firstName, lastName;
    private Calendar birthday;

    public Person(String fname, String lname, int month, int day, int year) {
        this.firstName = fname;
        this.lastName = lname;
        this.birthday = new GregorianCalendar(year, month-1, day);
    }

    public String toString() {
        return this.firstName + " " + this.lastName + ": " +
               (this.birthday.get(Calendar.MONTH)+1) + "/" +
               this.birthday.get(Calendar.DAY_OF_MONTH) + "/" +
               this.birthday.get(Calendar.YEAR);
    }

    ////////////////////////////////////////////////////////////////////////

    public static void main(String[] args) {
        Person p1 = new Person("Chris", "Marlowe", 5, 25, 1992);
        System.out.println(p1);
        System.out.println(p1.hashCode());

        Person p2 = new Person("Alex", "Cooper", 2, 5, 1994);
        System.out.println(p2);
        System.out.println(p2.hashCode());

        Person p3 = new Person("Pat", "Phillips", 2, 5, 1994);
        System.out.println(p3);
        System.out.println(p3.hashCode());

    }
}
```

```
run:
Chris Marlowe: 5/25/1992
424201356
Alex Cooper: 2/5/1994
2053965899
Pat Phillips: 2/5/1994
205238968
BUILD SUCCESSFUL (total time: 0 seconds)
```

a default hash function is defined for every `Object`

- uses *native code* to access & return the address of the object

overriding hashCode v.1

```
import java.util.Calendar;
import java.util.GregorianCalendar;

public class Person {
    private String firstName, lastName;
    private Calendar birthday;

    public Person(String fname, String lname, int month, int day, int year) {
        this.firstName = fname;
        this.lastName = lname;
        this.birthday = new GregorianCalendar(year, month-1, day);
    }

    public String toString() {
        return this.firstName + " " + this.lastName + ": " +
               (this.birthday.get(Calendar.MONTH)+1) + "/" +
               this.birthday.get(Calendar.DAY_OF_MONTH) + "/" +
               this.birthday.get(Calendar.YEAR);
    }

    public int hashCode() {
        return Math.abs((int)this.birthday.getTimeInMillis());
    }

    ////////////////////////////////////////////////////////////////////////

    public static void main(String[] args) {
        Person p1 = new Person("Chris", "Marlowe", 5, 25, 1992);
        System.out.println(p1);
        System.out.println(p1.hashCode());

        Person p2 = new Person("Alex", "Cooper", 2, 5, 1994);
        System.out.println(p2);
        System.out.println(p2.hashCode());

        Person p3 = new Person("Pat", "Phillips", 2, 5, 1994);
        System.out.println(p3);
        System.out.println(p3.hashCode());

    }
}
```

```
run:
Chris Marlowe: 5/25/1992
1899603840
Alex Cooper: 2/5/1994
218788608
Pat Phillips: 2/5/1994
218788608
```

can override hashCode if more class-specific knowledge helps

1. must consistently map the same object to the same index
2. must map equal objects to the same index

overriding hashCode v.2

```java
import java.util.Calendar;
import java.util.GregorianCalendar;

public class Person {
    private String firstName, lastName;
    private Calendar birthday;

    public Person(String fname, String lname, int month, int day, int year) {
        this.firstName = fname;
        this.lastName = lname;
        this.birthday = new GregorianCalendar(year, month-1, day);
    }

    public String toString() {
        return this.firstName + " " + this.lastName + ": " +
               (this.birthday.get(Calendar.MONTH)+1) + "/" +
               this.birthday.get(Calendar.DAY_OF_MONTH) + "/" +
               this.birthday.get(Calendar.YEAR);
    }

    public int hashCode() {
        return Math.abs((int)this.birthday.getTimeInMillis() +
                        (this.firstName+this.lastName).hashCode());
    }

    ////////////////////////////////////////////////////////////////////////

    public static void main(String[] args) {
        Person p1 = new Person("Chris", "Marlowe", 5, 25, 1992);
        System.out.println(p1);
        System.out.println(p1.hashCode());

        Person p2 = new Person("Alex", "Cooper", 2, 5, 1994);
        System.out.println(p2);
        System.out.println(p2.hashCode());

        Person p3 = new Person("Pat", "Phillips", 2, 5, 1994);
        System.out.println(p3);
        System.out.println(p3.hashCode());
    }
}
```

```
run:
Chris Marlowe: 5/25/1992
413568008
Alex Cooper: 2/5/1994
520715368
Pat Phillips: 2/5/1994
9438334
```

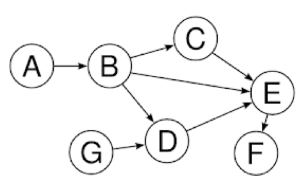
to avoid birthday collisions, can also incorporate the names

- utilize the String hashCode method

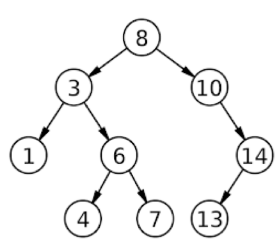
Graphs (sneak peek)

trees are special instances of the more general data structure: graphs

- informally, a graph is a collection of nodes/data elements with connections



a tree is a graph in which one node has no edges coming into it (the root) and no cycles

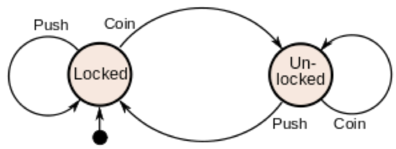


Finite State Machines (FSMs)

many useful problems can be defined using simple graphs

- a *Finite State Machine* (a.k.a. *Finite Automaton*) defines a finite set of states (i.e., nodes) along with transitions between those states (i.e., edges)

e.g., the logic controlling a coin-operated turnstile

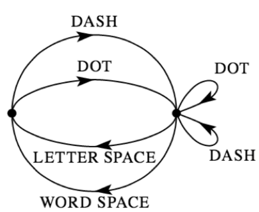


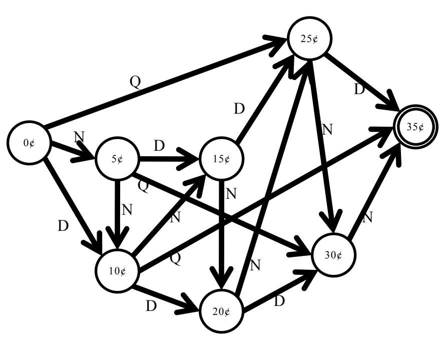
can be in one of two states: locked or unlocked

- if locked, pushing → it does not allow passage & stays locked
  inserting coin → unlocks it
- if unlocked, pushing → allows passage & then relocks
  inserting coin → keeps it unlocked

Other examples

Claude Shannon used a FSM to show constraints on Morse code

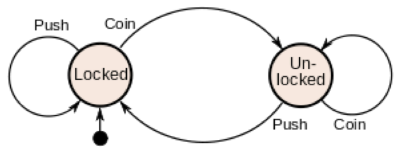




can use a FSM to specify the behavior of a vending machine

adding a coin (Q, D, N) changes the state

HW6: Simulate a FSM




```
locked push locked
locked coin unlocked
unlocked push locked
unlocked coin unlocked
```

model a FSM by storing the edges and providing lookup methods

```
private HashMap<StateLabel, HashMap<EdgeLabel, StateLabel>> table;
```

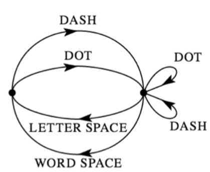
locked	**coin**	**unlocked**
	push	**locked**
unlocked	**coin**	**unlocked**
	push	**locked**

- the *key* to the table is the start state of an edge
- the *value* is another map, which maps edge labels to the end states

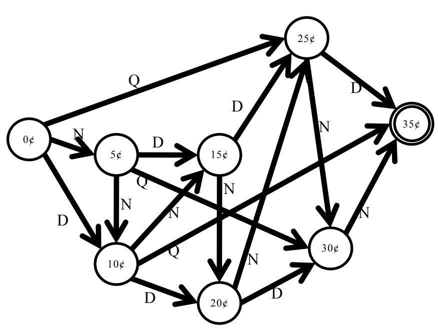
`table.get("locked")` →
a map containing edges from `"locked"`

`table.get("locked").get("coin")` →
`"unlocked"`

HW6: Other examples

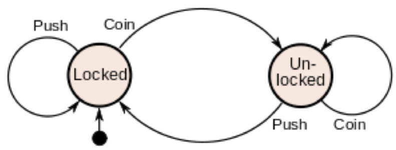



```
inLetter . inLetter
inLetter - inLetter
inLetter _ betweenLetters
inLetter __ betweenLetters
betweenLetters . inLetter
betweenLetters - inLetter
```




```
0cents N 5cents
0cents D 10cents
0cents Q 25cents
5cents N 10cents
5cents D 15cents
5cents Q 30cents
10cents N 15cents
10cents D 20cents
10cents Q 35cents
15cents N 20cents
15cents D 25cents
20cents N 25cents
20cents D 30cents
25cents N 30cents
25cents D 35cents
30cents N 35cents
```

HW6: PathTracer




```
locked push locked
locked coin unlocked
unlocked push locked
unlocked coin unlocked
```

given a start state and sequence of edges, determine the end state

```
Enter FSM file: turnstile.txt

Enter a start state (* to end): locked
Enter an edge sequence (separated with whitespace): coin push
End state: locked

Enter another start state (* to end): locked
Enter an edge sequence (separated with whitespace): push coin push push coin
End state: unlocked

Enter another start state (* to end): *
DONE
```


HW6: PathFinder

you are given a method that finds a shortest path between to states

```
fsm.findPath("0cents","35cents") → ["0cents","10cents","35cents"]
```

- you will write a driver class that repeatedly finds and prints paths

```
Enter FSM file: change.txt

Enter a start state (* to end): 0cents
Enter the end state: 35cents
State path: [0cents, 10cents, 35cents]

Enter another start state (* to end): 5cents
Enter the end state: 20cents
State path: [5cents, 10cents, 20cents]

Enter another start state (* to end): 10cents
Enter the end state: 5cents
No SUCH PATH

Enter another start state (* to end): *
DONE
```